

# JSA 2008-2009 Graduate Fellowship Final Report

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## 1 Introduction

I report on the work done during the academic year 2008-2009 under the assistance of the JSA Graduate Fellowship. My main research activity has been focused on the study of high energy scattering processes in gauge theories like Quantum Chromodynamics (QCD) and  $\mathcal{N} = 4$  Supersymmetric Yang-Mills (SYM) theory. During last decade it has become clear that non-linear effects become predominant in scattering processes at very high-energy (Regge) limit, and non-linear equations have been derived in order to describe the evolution of the structure of hadronic matter at this regime. One of these equations is the Balitsky-Kovchegov equation (BK) [1] derived first by Balitsky in 1996 in the Wilson lines formalism, and then by Kovchegov in 1999 in the dipole frame. The Wilson line formalism is an operator language which is based on the concept of factorization of the scattering amplitude in rapidity space and on the extension to high-energy (Regge) limit of the Operator Product Expansion (OPE) formalism, so far known only in the Bjorken limit as an expansion in terms of local operators or in terms of light ray operators. The relevance of the BK equation for future experiments like Electron Ion Collider (EIC) or Large electron Hadron Collider (LeHC) can be determined by including the running of the coupling constant and the evolution kernel to the next-leading-order (NLO) approximation in perturbative QCD (NLO corrections in power of strong coupling constant  $\alpha_s$ ). These are the motivations which made us to calculate the BK kernel at NLO approximation [2]. Such calculation allowed us to confirm the calculation of the NLO kernel [3] of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [4] as well. The BFKL equation can be seen as the linear version of the BK equation, but which does not include non-linear effects present at very high-energy and which also violates the unitarity condition preserved instead by the BK equation. Conformal symmetry is a symmetry violated in QCD by the running of the coupling constant. What one would then expect from the calculation of the NLO BK-kernel is that the only source of violation of such symmetry come from the running of coupling while the rest of the kernel preserve conformal invariance. However, although Wilson lines are formally conformal invariant they are divergent, and since at the moment it is still not known how to regularize them in a conformal invariant way, in the NLO-BK kernel calculation there are non conformal terms as remnant of the prescription used to cure such divergences. In order to study the loss of conformal invariance we considered a conformal invariant theory such as the  $\mathcal{N}=4$  supersymmetric Yang-Mills (SYM) theory and we calculated the NLO-BK kernel [5] in this framework. It turned out that suitable operators for the description of processes at high-energy

(Regge) theory are composite conformal (Wilson line) operators which can be constructed order by order in perturbation theory. These operators absorb the undesired non conformal terms in the same way as counterterms are added to renormalize local composite operators in order to restore the symmetry that the bare operator lost at the level of NLO (and higher) corrections. The NLO BFKL kernel is also known in  $\mathcal{N}=4$  SYM theory[6]; in our calculation we confirmed that result and calculated for the first time the conformal NLO BFKL kernel [7]. As an example of the validity of the operator product expansion in terms of conformal composite Wilson line operator and as a proof of the validity of the factorization in rapidity at the NLO level, we calculated the NLO amplitude of a four-point function made of renorm-invariant operator in  $\mathcal{N}=4$  SYM theory [8]. The result is that we confirmed the known Pomeron intercept at small  $\alpha_s$  at NLO, and also obtained the Pomeron residue to the NLO and an analytic expression for the NLO amplitude.

## 2 Activity during the academic year 2008-2009

### 2.1 Publications

- I. Balitsky and G. A. Chirilli, “NLO evolution of color dipoles in N=4 SYM,”  
Nucl. Phys. B **822**, 45 (2009) [arXiv:0903.5326 [hep-ph]].
- I. Balitsky and G. A. Chirilli, “Conformal kernel for NLO BFKL equation in  $\mathcal{N}=4$  SYM,”  
Phys. Rev. D **79**, 031502 (2009) [arXiv:0812.3416 [hep-ph]].
- I. Balitsky and G. A. Chirilli, “Next-To-Leading Order Evolution Of Color Dipoles,”  
Phys. Rev. D **77**, 014019 (2008) [arXiv:0710.4330 [hep-ph]].

### 2.2 Proceedings

- G. A. Chirilli “High-Energy Operator Product Expansion at NLO”  
to be published in *Modern Physics Letters A*.
- G. A. Chirilli, “NLO Evolution of Color Dipoles in N = 4 SYM,”  
AIP Conf. Proc. **1105**, 331 (2009) [arXiv:0811.3642 [hep-ph]].
- I. Balitsky and G. A. Chirilli, “NLO evolution of color dipoles,”  
Acta Phys. Polon. Supp. **1**, 545 (2008).
- I. Balitsky and G. A. Chirilli, “NLO evolution of color dipole,”  
Acta Phys. Polon. B **39**, 2561 (2008).

## 3 Invited talk

- June 26, 2009 *Brookhaven National Laboratory* (New York, USA)  
– **Title talk:** NLO evolution of Color Dipole in QCD and  $\mathcal{N} = 4$  SYM
- January 6, 2009 *Laboratoire de Physique Théorique* (Paris, France)  
– **Title talk:** NLO evolution of Color Dipole in QCD and  $\mathcal{N} = 4$  SYM

### 3.1 Conferences and Workshops

- 08-14 Sep, 2008 *International Workshop on Diffraction in High-Energy Physics (Diffraction 2008)*, (La Londe-les-Maures, France)
  - **Title talk:** NLO Evolution of Color Dipole in QCD and  $\mathcal{N} = 4$  SYM
- 14-18 Jul, 2008 *Structure of Hadrons and Nuclei at an Electron Ion Collider* (Trento, Italy)
  - **Title talk:** NLO evolution of Color Dipole in QCD and  $\mathcal{N} = 4$  SYM
- 06-10 Jul, 2008 *Low- $x$  meeting workshop* (Kolimpari, Crete, Greece)
  - **Title talk:** Next-to-leading Order Evolution of Color Dipole
- 19-23 May, 2008 *4th Electron-Ion Collider Workshop* (Hampton University, Hampton, USA)
  - **Title talk:** Next-to-leading Order Evolution of Color Dipole

### 3.2 Summer school

- 2008** : School on Hadronic collision at the LHC and QCD at high density in Les Houches (France)
- **Title talk:** *Next-to-leading Order Evolution of Color Dipole*

## 4 Acknowledgments

I am very grateful to the JSA Graduate fellowship for the support I received during my last academic year of my PhD program. The travel opportunities the fellowship provided me were extremely important for my professional career. I had the opportunity to present our results at several conferences and to interact with other people, to discuss about my research and about future possible research projects. I can certainly say that these travel opportunities helped me a lot in finding my new position as a post doctorate researcher at Ecole Polytechnique in Paris.

## References

- [1] I. Balitsky, *Nucl. Phys.* **B463**, 99 (1996); [arXiv:9706411 [hep-ph]]; Yu.V. Kovchegov, *Phys. Rev.* **D60**, 034008 (1999), *Phys. Rev.* **D61**,074018 (2000).
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[arXiv:0812.3416 [hep-ph]].
- [8] I. Balitsky and G. A. Chirilli, [arXiv:0911.5192 [hep-ph]].