Extraction of light meson radiative transition matrix elements from lattice QCD

A research report prepared for the 2012-13 JSA Graduate Fellowship Program.

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I Research

This report details research progress made during the 2012-13 academic year while supported by the Jefferson Science Associates Graduate Fellowship Program. During this time I developed and tested a suite of analysis tools necessary to extract transition matrix elements from lattice QCD three-point functions. These matrix elements, evaluated for real photons, drive the rate of photoproduction of mesons off nuclear targets and are of particular relevance to the upcoming GlueX experiment sited in Hall D. GlueX aims to comprehensively investigate the spectrum of light quark mesons paying particular attention to those with exotic quantum numbers. Observation of a spectrum of hybrids promises to shed light on the effect of gluonic degrees of freedom. Here the aim is to lay the groundwork for precision calculations with which to confront future experimental measurements and rigorously test our description of the strong force.

The remainder of this report is as follows. Section II lays out the technology used to extract transition form factors from lQCD three-point functions. Preliminary results using a distilled isovector insertion in place of the actual photon are presented in Section III. Section IV details spectroscopic work currently in progress.

II Three Point Functions

The aim of this work is to extract matrix elements between hadron states $|i\rangle$ and $|f\rangle$ of the form $\langle f | j^\mu | i \rangle$ where $j^\mu$ is the vector current insertion corresponding to the hadronic structure of the photon. On the lattice the field theoretic quantity that encodes radiative transitions is the three-point function

$$ C^\mu(p_f, p_i, t_f, t_i) = \langle \Omega_f(t, p_f) \bar{\psi} \gamma^\mu \psi(t, q) \Omega_i^\dagger(t_i, p_i) \rangle $$

(1)

The $\Omega_{i,f}$ are variationally optimized operators constructed is such a way as to interpolate only a single eigenstate of the transfer matrix. These operators are projected onto definite spatial momentum, $p_f, p_i, q \equiv p_f - p_i$, via a discrete Fourier transform summing over lattice sites with weight $e^{\pm i\vec{p} \cdot \vec{x}}$. The three point functions are related to the meson matrix elements via inserting a complete set of states and performing Euclidean time evolution. The result is a propagation factor $P = e^{-E_f(t_f-t) - E_i(t-t_i) \sum Z_f^* Z_i} \frac{Z_f Z_i}{E_f E_i}$ times the matrix element we want to isolate.
\[ C^{\mu}(p_f, p_i, t_f, t_i) = P(p_f, p_i, t_f, t_i)\langle f(p_f)|j^{\mu}(q)|i(p_i) \rangle \]  

(2)

Here the energies (E) and operator-state overlaps (Z) of a given state are obtained via a two point function analysis [4]. The matrix elements are then decomposed into a known kinematic factor (e.g. \( K^{\mu} = p_f^{\mu} + p_i^{\mu} \) for \( \langle \pi(p_f)|j^{\mu}|\pi(p_i) \rangle \)) multiplying an unknown form factor (e.g. \( F_{\pi}(Q^2) \)). One can then form a system of equations involving rotationally equivalent momentum combinations and different combinations of Lorentz indices (labelled by a,b,c..) to define a linear system,

\[
\begin{pmatrix}
C^a \\
C^b \\
C^c \\
\vdots
\end{pmatrix} = \begin{pmatrix}
P(a, t)K^a_1 & P(a, t)K^a_2 & \ldots \\
P(b, t)K^b_1 & P(b, t)K^b_2 & \ldots \\
P(c, t)K^c_1 & P(c, t)K^c_2 & \ldots \\
\vdots & \vdots & \ddots
\end{pmatrix} \begin{pmatrix}
F_1(Q^2) \\
F_2(Q^2) \\
\vdots
\end{pmatrix}
\]

which we then can invert via SVD to solve for the form-factors (\( F_n(Q^2) \)).

III Distilled Iso-Vector Pion Form Factor

Using the distillation framework laid out in [1] we were successful in performing a “proof of principle” calculation that extracted the iso-vector pion form-factor. The plot below is the pion iso-vector form-factor using a distillation smeared current calculated at two different pion masses using the Hadron Spectrum Collaboration’s 2+1 anisotropic clover lattices. In order to show both graphs on the same plot we postulated multiplicative renormalizability of the form factor which not a true property of the calculation. Using a distillation smeared insertion in fact breaks the Ward Identity and introduces a momentum dependent renormalization constant evidenced by the “scatter” of points in the plot below. A calculation involving an unsmeared insertion is presently under way.

![Figure 1: \( F_{\pi}(Q^2) \) extracted at two different pion masses using a distilled (no Ward-Identity) photon. Red points correspond to \( m_\pi \sim 700 \text{MeV} \), blue to \( m_\pi \sim 400 \text{MeV} \).](image-url)
IV Current Work

As mentioned above, using a distillation smeared insertion breaks the Ward Identity and introduces a momentum dependent renormalization constant. This is further complicated by the fact that we are using anisotropic lattices and have also explicitly broken O(4) hyper-cubic symmetry. Calculations are presently under way to construct correlation functions using an unsmeared vector current insertion.

Such a calculation involves construction of a new set of three point correlation functions. This entails computing quark propagators on background gluonic fields and tying them together in such a way as to avoid a distillation index occurring in the insertion. The procedure eventually boils down to a series of large linear algebra problems that can be tackled efficiently using the HPC infrastructure located at Jefferson Lab.

V Travel

Travel funding provided by the JSA fellowship was used to attend the New Horizons in Lattice Field Theory workshop in Natal, Brazil. The meeting consisted of short courses and tutorials on fundamental topics taught by prominent members of the field and was an extremely educational and rewarding experience.

References


