Extraction of light meson radiative transition matrix elements from lattice QCD

Christian J. Shultz$^{1,}$*

$^1$Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

(Dated: January 28, 2015)

A research report prepared for the 2012-13 JSA Graduate Fellowship Program under the supervision of Jozef J. Dudek.

I. RESEARCH

This report details research progress made during the 2013-14 academic year while supported by the Jefferson Science Associates Graduate Fellowship Program. During this time I developed and tested a suite of analysis tools necessary to extract transition matrix elements from lattice QCD three-point functions. These matrix elements, evaluated for real photons, drive the rate of photoproduction of mesons off nuclear targets and are of particular relevance to the upcoming GlueX experiment sited in Hall D. GlueX aims to comprehensively investigate the spectrum of light quark mesons paying particular attention to those with exotic quantum numbers. Observation of such a spectrum promises to shed light on the effect of gluonic degrees of freedom.

II. MOTIVATION

Mesons in particular serve as an ideal meeting ground between theory and experiment. Experimentally the spin and parity distribution of states in conjunction with the lack of states with strangeness or isospin greater than one indicates that mesons might be described, in a minimal context, by simply coupling together a constituent quark and antiquark into an object of spin $S = 0, 1$. Inclusion of orbital angular momentum allows for a prediction of multiplets of states, based on quantum mechanics angular momentum addition rules, in terms of their spin, parity, and charge conjugation quantum numbers $J^{PC}$.

Conspicuously absent from this picture is the gluonic contribution. Short of transforming bare quarks into constituent quarks it seems to play no role in the spectrum. This is unexpected, QCD is a strongly coupled gauge theory. The simple $qar{q}$ should not be the entire story, indeed even in the absence of quarks, pure gauge theory, gluons bind together forming states composed entirely of glue, glueballs.

The hybrid sector provides a promising testing ground of the hidden gluonic contributions to the spectrum of QCD. Provided the gluonic field excitation has quantum numbers other than $0^{++}$ we can generate $J^{PC}$ outside of the set allowed in a quark-antiquark picture, for example $J^{PC} = 0^{-+}, 0^{+-}, 1^{--}, 2^{++}$. These quantum numbers are known as exotic and are one of the best signature for hadronic physics extending beyond the constituent quark picture. To date there has been no unambiguous experimental observation of mesons having any such quantum numbers.

The goal of this project was to make inroads towards non-perturbative calculation of the hybrid photocouplings relevant to GlueX. The tool we propose to use to investigate these transition matrix elements is lattice QCD, a first principles based numeric approach to estimating correlation functions, theoretical quantities encoding the dynamics of QCD, based on discretizing the theory on a finite grid of Euclidean space-time points. Correlation functions are evaluated over a large but finite number of gauge configurations providing for a systematically improvable framework within which we can obtain information about the non-perturbative dynamics of Quantum Chromodynamics.

In this text we will reproduce the first set of results, form-factors and transitions between pseudoscalar and vector members of SU(3)$_F$ octet. This is the first demonstration of the set of techniques we propose to use to extract the photo-couplings of exotic and hybrid mesons at the SU(3)$_F$ point, results relevant to the ongoing GlueX experiment cited in Hall D at Jefferson Lab. These results are a summary of a preprint soon to be submitted for publication.

III. THE CALCULATION

The calculation we performed relies on first extracting spectroscopic information, in the relevant channels of quantum numbers, via the calculation of a matrix of correlation functions over a large basis of operators, $\{O_i\}$. Akin to the Rayleigh-Ritz method in quantum mechanics we propose that there is a linear superposition of operators within the basis that most best creates each state in the spectrum, the different superpositions being orthogonal in a suitable sense. Identification of these linear combinations follows from application of the variational method to lattice two-point correlation functions. In Figure 1 we reproduce the identified spectrum of states, the central panel corresponds to transitions that we have successfully extracted. Further details about variational spectroscopy can be found in [1-3].

Having identified the optimal superpositions of operators, for each state in our spectrum, both at rest and in flight, we can proceed to use these creation and an-
nihilation operators to calculate three-point correlation functions featuring the vector current. These correlation functions encode the transition amplitude for a meson eigenstate $|h\rangle$ to decay, radiatively, to another $|h'\rangle$. In the case that the initial and final states are the same we instead speak of form-factors whose dependence on photon virtuality can be related to the distribution of quarks within the hadron.

The photon couples to the electric charges of $u, d, s$ quarks via the vector current $j^\mu = \frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d - \frac{1}{3} \bar{s} \gamma^\mu s$, up to a factor of the magnitude of the electron charge, $e$. In general a transition induced by this current between a hadron, $h$, of spin-$J$ and a hadron $h'$ of spin-$J'$ is described by the matrix element,

$$\langle h' \gamma^\mu (\lambda', \bar{p}') | j^\mu | h (\lambda, \bar{p}) \rangle,$$

where the spin-state of $h$ is specified in terms of its helicity, $\lambda$, the projection of $\vec{J}$ along the direction of momentum $\bar{p}$. These matrix elements are simply related to the helicity amplitude for the transition $\gamma h \to h'$ by including the initial-state photon’s polarization vector where the photon has a virtuality $Q^2 = -q^2 = |\bar{p}' - \bar{p}|^2 - (E_{h'}(\bar{p}') - E_h(\bar{p}))^2$.

There are relations between these amplitudes which follow from the constraints of Lorentz invariance, current conservation and invariance under parity transformations. These can be accounted for if we write a matrix-element decomposition in terms of a number of Lorentz invariant form-factors, $F_i(Q^2)$. For any given pair of mesons $h, h'$, of definite spin and parity, there are only a limited number of possible constructions consistent with parity invariance and with the additional constraint of current conservation, we can write explicit decompositions in terms of a few independent form-factors.

For example, the transition matrix-element between a vector particle and a pseudoscalar can be expressed as

$$\langle \pi^+(\bar{p}') | j^\mu (\lambda, \bar{p}) \rangle = \epsilon^{\mu\nu\rho\sigma} p'_\nu p_\rho \epsilon_\sigma (\lambda, \bar{p}) \frac{2}{m_\pi + m_\rho} F_{\rho\pi}(Q^2),$$

and for a vector meson stable under the strong interactions, the transition form-factor at $Q^2 = 0$ can be related to the radiative decay width $\Gamma(\rho^+ \to \pi^+ \gamma) = \frac{3}{2} \alpha \frac{2}{m_\pi + m_\rho} |F_{\rho\pi}(0)|^2$, where $q$ is the momentum of the final-state photon in the rest-frame of the decaying $\rho$ meson.

In Figure 2 we reproduce the $\rho \to \pi\gamma$ radiative transition form-factor for a large number of values of $Q^2$. Such a large number of measurements allows us to interpolate between the timelike and spacelike regions of $Q^2$ in order to extract a value for the photocoupling which controls...
FIG. 2. Ground-state $\rho$ to ground-state $\pi$ transition form-factor. Curves in gray show fits used to interpolate between spacelike and timelike regions to determine the photocoupling, $F_{\rho\pi}(0)$. Experimental decay widths converted to photocouplings shown for orientation.

the strength of the partial decay width.

The calculation performed here uses three degenerate quark flavors tuned to approximate the physical strange quark mass and as such our photocoupling determination cannot be directly compared with experiment. For orientation we show in Figure 2, the experimental values, $F_{\rho\pi}(0) = 0.33(2)$ and $F_{K^*K^*}(0) = 0.57(3)$ extracted from the corresponding decay rates obtained via the Primakoff effect for pions and kaons incident on nuclear targets [4–6].

This methods demonstrated in this work also provide partial resolution to the problem of extraction of excited state matrix elements from lattice calculations. These excited state matrix elements appear as sub-leading exponentially suppressed contributions to correlation functions. Formation of optimized operators directly deals with this problem by allowing us to compute correlators whose principal time dependence is not the ground state but rather the excited states of interest. Isolation of these excited state signals in turn allows us to robustly compute quantities, from first principles on the lattice, whose extraction had previously been plagued by systematics associated with extraction of suppressed exponentially damped signals. A preprint, demonstrating the efficacy of these analysis methods, is complete and will be submitted for publication shortly.

IV. FUTURE DIRECTIONS

This first study we performed was restricted to transitions involving pseudoscalar and vector mesons but may easily be extended to other quantum numbers, in particular the exotic sector. The methods also allow for extension to the baryon sector. Here the $Q^2$ dependence can be measured quite directly via electro-production of excited nucleons of proton and neutron targets. The relative magnitudes of the multipole elements is one possible means to access the internal quark-gluon structure of $N^*$ and $\Delta^*$ states [7]. Calculations are also presently underway to extract the $\rho \rightarrow \pi \gamma$ transition amplitude at a lighter pion mass where the $\rho$ meson appears as a dynamically generated resonance in $\pi \pi$ scattering.


V. TALKS

Union College, Schenectady, New York Hadronic Physics and lQCD September 2013

University of Adelaide, Adelaide, Australia Radiative Physics on the Lattice January 2014

Old Dominion University, Norfolk, Virginia lattice QCD and Hadronic Spectroscopy February 2014

Hadron Spectrum Collaboration Meeting, Newport News, Virginia Formfactors and Transitions May 2014

Lattice 2014, New York, New York Radiative Physics from Lattice QCD using Distillation June 2014

VI. TRAVEL

I received partial travel support to give a talk at the University of Adelaide, Adelaide Australia, where I demonstrated the efficacy of the novel methods we apply to matrix element calculations as well as interact with their Nuclear and Particle physics Theory Group.