The Proton Spin-dependent Structure $g_2$ at Low $Q^2$

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Outline

• Introduction
   (Refer to A. Deur’s talk “nuclear spin structure study at Jlab”, Session Helicity-Parallel II)

• Physics Motivation
   (Refer to K. Slifer’s talk “nucleon spin structure with lepton beam at low $Q^2$”, Plenary X)

• Experiment Setup

• Analysis and Preliminary Results
Electron Scattering

Important kinematics variables:

- $\nu = E - E'$
- $Q^2$: Momentum transfer squared
- $W$: Invariant mass of residual hadronic system
- $x = \frac{Q^2}{2M\nu}$: Bjorken variable: fraction momentum of struck quark

Diagram:

- Incident electrons $E$
- Target
- Scattered electrons $E'$
- Detectors
- Hadronic final states

Mathematical expressions:

- $k = (E, \vec{k})$
- $k' = (E', \vec{k}')$
- $q = (\nu, \vec{q})$
- $P = (M, \vec{0})$
- $P' = (E'_t, \vec{P}')$
Electron Scattering

- Inclusive *unpolarized* cross section:

\[
\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right]
\]

Structure Function which indicates the parton distribution
If the beam and target are polarized, the asymmetric part of the lepton and hadron tensor will not vanish, which leads to 2 additional structure functions $g_1$ and $g_2$.

\[
\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} + \gamma g_1(x, Q^2) + \delta g_2(x, Q^2) \right]
\]

2 additional Structure Function which related to the polarized parton distribution.
Spin Structure Function

- At Bjorken limit, $g_1$ related to the polarized parton distribution functions

$$g_1 = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \quad \Delta q_i(x) = q_i^\uparrow(x) - q_i^\downarrow(x)$$

- $g_2$ is zero in the naive parton model: non-zero value carries information of quark-gluon interaction

- Concept of “twist”:
  - Leading twist: related to amplitude for scattering off asymptotically free quarks
  - Higher twists: quark-gluon interaction and the quark mass effects
Spin Structure Function

- $g_2^{WW}$ is the leading twist part of the $g_2$:
  $$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \tilde{g}_2(x, Q^2)$$
- which can be calculated from $g_1$ with the Wandzura-Wilczek relation
  $$g_2^{WW} = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2)$$
- Higher twist components can be expressed as:
  $$\tilde{g}_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y} \left[ \frac{m_q}{M} h_T(y, Q^2) + \zeta(y, Q^2) \right] \frac{dy}{y}$$

  - quark transverse momentum contribution
  - twist-3 part which arises from quark-gluon interactions
- Will get information about higher twist effect when measuring $g_2$
How to get \( g_2 \)

\[
\Delta \sigma_{||} = \begin{array}{c}
\text{e}^- \\
\hline
\text{p} \\
\hline
\text{e}^-
\end{array} - \begin{array}{c}
\text{e}^- \\
\hline
\text{p} \\
\hline
\text{e}^-
\end{array}
\]

JLab Hall B experiment EG4 measured this quantity

\[
\Delta \sigma_{||} = \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\uparrow}}{d\Omega dE'} = \frac{4\alpha^2 E'}{M\nu Q^2 E} \left[ (E + E' \cos \theta) g_1 - 2M_x g_2 \right]
\]

\[
\Delta \sigma_{\perp} = \begin{array}{c}
\text{e}^- \\
\hline
\uparrow \text{p} \\
\hline
\downarrow \text{e}^-
\end{array} - \begin{array}{c}
\text{e}^- \\
\hline
\uparrow \text{p} \\
\hline
\downarrow \text{e}^-
\end{array}
\]

\[
\Delta \sigma_{\perp} = \frac{d^2\sigma^{\uparrow\rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\rightarrow}}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{M\nu Q^2 E} \sin \theta \left[ g_1 + \frac{2E}{\nu} g_2 \right]
\]

\( g_2^p \) experiment will measure this, combing the EG4 data to get \( g_2^p \) at low \( Q^2 \)
Physics Motivation

- Measure the proton structure function $g_2$ in the low $Q^2$ region (0.02-0.2GeV$^2$) for the first time
  - Extract the generalized longitudinal-transverse spin polarizability $\delta_{LT}$ as a test of Chiral Perturbation Theory ($\chi$PT) calculations
  - Test the Burkhardt-Cottingham (BC) sum rule
  - Crucial inputs for Hydrogen hyperfine splitting calculation
SLAC experiment E143, E155, E155x and JLab experiment RSS and SANE have measured proton $g_2$ on a wide $Q^2$ range

- However lack low $Q^2$ data

Existing Data

$Q^2 \sim 5 \text{ GeV}^2$

SLAC

$Q^2 = 2.5 \sim 5.5 \text{ GeV}^2$

JLab SANE

$Q^2 = 1.3 \text{ GeV}^2$

JLab RSS

• SLAC experiment E143, E155, E155x and JLab experiment RSS and SANE have measured proton $g_2$ on a wide $Q^2$ range

• However lack low $Q^2$ data
Generalized Longitudinal-Transverse Polarizability

- From the dispersion relation of the doubly-virtual Compton scattering amplitude, one could derive generalized spin polarizability

\[
\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right] dx
\]

\[
\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 + g_2 \right] dx
\]

- Can be expressed as structure functions

- Can be calculated via Chiral Perturbation Theory

Neutron data shows large deviation between data and $\chi$PT prediction

Generalized Longitudinal-Transverse Polarizability

• At low $Q^2$, the generalized polarizabilities have been evaluated with NLO $\chi$PT calculations:
  
  

• One issue in the calculation is how to properly include the nucleon resonance contributions, especially the $\Delta$ resonance
  
  • $\gamma_0$ is sensitive to resonances
  
  • $\delta_{LT}$ is insensitive to the $\Delta$ resonance

  • $\delta_{LT}$ should be more suitable than $\gamma_0$ to serve as a testing ground for the chiral dynamics of QCD
Generalized Longitudinal-Transverse Polarizability

- Improved calculation result with Relativistic Baryon χPT:

![Graph showing the comparison of proton and neutron polarizability](image)

The neutron data point are from E94-010

- Red solid line: LO
- Blue band: NLO
- Black dashed line: MAID model

Generalized Longitudinal-Transverse Polarizability

- Improved calculation result with Relativistic Baryon $\chi$PT:

\begin{itemize}
  \item It was claimed that the $\delta_{LT}$ puzzle is solved with this new calculation, however it should be tested with proton data.
\end{itemize}

BC Sum Rule

- **BC Sum Rule:**
  \[ \int_0^1 g_2(x, Q^2) \, dx = 0 \]

- Violation suggested for proton at large $Q^2$
- But found satisfied for the neutron
- Mostly unmeasured for proton
- To experiment test BC sum rule, one need to combine measured $g_2$ data with some low $x$ model and elastic contribution

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SLAC E155x
- Hall C RSS
- Hall A E94-010
- Hall A E97-110 (preliminary)
- Hall A E01-012 (preliminary)
Proton Radius Puzzle

- The finite size of the nucleus plays a small but significant role in atomic energy levels
- Simplest: proton
- 2 ways to measure:
  - energy splitting of the $2S_{1/2}-2P_{1/2}$ level (Lamb shift)
  - scattering experiment
- The result do not match when using muonic hydrogen
  - $\langle R_p \rangle = 0.84184 \pm 0.00067 \text{fm}$ by Lamb shift in muonic hydrogen
  - $\langle R_p \rangle = 0.87680 \pm 0.0069 \text{fm}$ CODATA world average

Hydrogen Hyperfine Structure

- Hydrogen hyperfine splitting in the ground state has been measured to a relative high accuracy of $10^{-15}$

  \[
  \Delta E = 1420.4057517667(9) \text{MHz} \\
  = (1 + \delta)E_F \\
  \delta = (\delta_{\text{QED}} + \delta_{R} + \delta_{\text{small}}) + \Delta_S
  \]

- $\Delta_S$ is the proton structure correction and has the largest uncertainty

  \[
  \Delta S = \Delta_Z + \Delta_{\text{pol}}
  \]

- $\Delta_Z$ can be determined from elastic scattering, which is $-41.0 \pm 0.5 \times 10^{-6}$

- $\Delta_{\text{pol}}$ involves contributions of the inelastic part (excited state), and can be extracted to 2 terms corresponding to 2 different spin-dependent structure function of proton
Hydrogen Hyperfine Structure

\[ \Delta_{\text{pol}} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2) \]

\[ \Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2) \]

\[ B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2) \]

\[ \beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)} \]

- \( B_2 \) is dominated by low \( Q^2 \) part
- \( g_2^p \) is unknown in this region, so there may be huge error when calculating \( \Delta_2 \)
- This experiment will provide a constraint

Experiment Setup

g2p experiment ran in Jefferson Lab Hall A from Feb 29th to May 18th, 2012
Experiment Setup

- Major new installed instruments in Hall A
  - Polarized NH$_3$ target
  - Low current beam diagnostics
  - Septa magnets

Beam diagnostics:

- Polarized NH$_3$ Target
- Local Dump
- Septa magnets

Hall A High Resolution Spectrometer (HRS)
Polarized Target

- Polarized NH3 Target
  - 2.5T/5.0T field generated by a pair of Helmholtz coils for polarizing solid NH3 target material
  - Outgoing beam will be tilted by the large target field
Kinematics Coverage

\[ M_p < W < 2 \text{ GeV} \]
\[ 0.02 < Q^2 < 0.2 \text{ GeV}^2 \]

<table>
<thead>
<tr>
<th>Beam Energy (GeV)</th>
<th>Target Field (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.254</td>
<td>2.5</td>
</tr>
<tr>
<td>1.706</td>
<td>2.5</td>
</tr>
<tr>
<td>1.158</td>
<td>2.5</td>
</tr>
<tr>
<td>2.254</td>
<td>5</td>
</tr>
<tr>
<td>3.352</td>
<td>5</td>
</tr>
</tbody>
</table>

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Analysis

\[ A_{\text{phy}}^{\text{phys}} = \frac{A_{\perp}^{\text{raw}}}{D P_b P_t} \]

\[ A_{\perp}^{\text{raw}} = \frac{N^+}{Q^+} - \frac{N^-}{Q^-} = \frac{N^+}{Q^+} + \frac{N^-}{Q^-} \]

Dilution factor (finished)

Beam and target polarization (finished)

Charge and yield in different beam helicity state (finished)

\[ \sigma_{0}^{\text{phys}} = \sigma_{0}^{\text{raw}} \times D \]

\[ \sigma_{0}^{\text{raw}} = \frac{N}{N_{\text{in}} \rho \varepsilon_{\text{det}}} \times \frac{1}{A} \]

Total Charge (finished)

Target Density

Detector Efficiency (finished)

Acceptance (on-going)

- Subjects as input:
  - Beam position
  - Spectrometer optics

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Optics Study

- HRS has a series of magnets
  - 3 quadrupoles to focus and 1 dipole to disperse on momentums
- Optics study will provide a matrix to transform VDC readouts to kinematics variables which represents the effects of these magnets

\[
\begin{pmatrix}
\delta \\
\theta \\
y \\
\phi
\end{pmatrix}
t_{g} =
\begin{pmatrix}
\langle \delta | x \rangle & \langle \delta | \theta \rangle \\
\langle \theta | x \rangle & \langle \theta | \theta \rangle \\
\langle y | y \rangle & \langle y | \phi \rangle \\
\langle \phi | y \rangle & \langle \phi | \phi \rangle
\end{pmatrix}
\begin{pmatrix}
x \\
\theta \\
y \\
\phi
\end{pmatrix}
\]
Optics Study

- Optics study for g2p: the most important part is how to treat the transverse target field.

- Idea: separate reconstruction process to 2 parts:
  - Use the normal optics matrix to reconstruct from the VDC to sieve slit.
  - Use the field map to do ray tracing of the scattered electrons from sieve slit to target.
Optics Calibration

- Run simulation to decide the effective theta and phi
  - Use the BPM readout to set the beam position
  - Beam energy 1.706 GeV, target field 2.5T

Initial scattering angle

Effective angle to do the fitting

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Optics Calibration: Angle

Resolution: 1.6 mrad (RMS)

LHRS

Before Calibration

After Calibration

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Optics Study

- The performance summary of the optics with target field: the table shows a summary of the RMS values of each kinematic variables after calibration

<table>
<thead>
<tr>
<th>HRS</th>
<th>Beam Energy (GeV)</th>
<th>Filed Strength (T)</th>
<th>Filed Angle (deg)</th>
<th>$\delta$</th>
<th>$\theta$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>2.254</td>
<td>2.5</td>
<td>90</td>
<td>2.2x10^-4</td>
<td>1.8 mrad</td>
<td>1.8 mrad</td>
</tr>
<tr>
<td>L</td>
<td>1.710</td>
<td>2.5</td>
<td>90</td>
<td>2.4x10^-4</td>
<td>2.4 mrad</td>
<td>1.5 mrad</td>
</tr>
<tr>
<td>L</td>
<td>1.157</td>
<td>2.5</td>
<td>90</td>
<td>3.2x10^-4</td>
<td>2.1 mrad</td>
<td>1.3 mrad</td>
</tr>
<tr>
<td>L</td>
<td>2.254</td>
<td>5.0</td>
<td>0</td>
<td>2.2x10^-4</td>
<td>1.6 mrad</td>
<td>1.2 mrad</td>
</tr>
<tr>
<td>R</td>
<td>2.254</td>
<td>2.5</td>
<td>90</td>
<td>2.5x10^-4</td>
<td>2.2 mrad</td>
<td>1.8 mrad</td>
</tr>
<tr>
<td>R</td>
<td>1.710</td>
<td>2.5</td>
<td>90</td>
<td>2.3x10^-4</td>
<td>2.7 mrad</td>
<td>1.7 mrad</td>
</tr>
<tr>
<td>R</td>
<td>1.157</td>
<td>2.5</td>
<td>90</td>
<td>3.4x10^-4</td>
<td>1.9 mrad</td>
<td>1.5 mrad</td>
</tr>
</tbody>
</table>

- The optics with target field works well

Thanks to Chao Gu, Min Huang
Preliminary Results

- Fully radiated asymmetries (red curve)
- Cross section models: P. Bosted's fit (unpolarized) and MAID 2007 (polarized)
- Include Unpolarized and polarized elastic tail
- Radiating methods: Mo/Tsai (unpolarized) and Akushevich/Ilyichev/Shumeiko (polarized)

\[ A_\perp = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\Rightarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\downarrow\Rightarrow}} \]
\[ A_\parallel = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\downarrow\uparrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\downarrow\uparrow}} \]
Preliminary Results

- Preliminary results for 2.254GeV, 5T trans configuration
- The unpolarized cross section is from P. Bosted's fit
- Compared with radiated MAID model prediction

\[ \Delta \sigma_\perp = A_\perp \times \sigma_0 \]

\[ A_\perp = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\Rightarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\downarrow\Rightarrow}} \]
Preliminary Results

- Preliminary results for 2.254 GeV, 5.0T trans configuration
- $Q^2 \sim 0.1 \text{ GeV}^2$ for this setting
- The integral is from $x=0$ to the pion threshold
- We measured $x$ as low as 0.04 and the unmeasured region will be evaluated with $g_2^{WW}$

\[ \delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] \, dx \]

Integrand of $\delta_{LT}$

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Preliminary Results

- Low $x$ contribution is suppressed due to the $x^2$ weight in the integral.

- Once the analysis is done, we should be able to provide the result at four different $Q^2$ as shown in the plot.

\[
\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx
\]
The g2p experiment ran in spring 2012 and took data covering $0.02 < Q^2 < 0.20 \text{ GeV}^2$

- Will provide an accurate measurement of $g_2$ in low $Q^2$ region for the first time
  - Extract the fundamental quantities $\delta_{LT}$ to provide a test of $\chi$PT calculations
  - Test the Burkhardt-Cottingham (BC) Sum Rule
- New instruments are demonstrated working well during the experiment (1 NIM paper published and 1 NIM paper in preparation)
- Data analysis is currently underway
g2p Collaboration

Spokespeople
Alexander Camsonne
J.P. Chen
Don Crabb
Karl Slifer

Post Docs
Kalyan Allada
Elena Long
Vince Sulkosky
Jixie Zhang

Graduate Students
Toby Badman
Melissa Cummings
Chao Gu
Min Huang
Jie Liu
Pengjia Zhu
Ryan Zielinski
Backups
Analysis

- To reduce uncertainty, polarized cross section difference is derived from asymmetry and unpolarized cross section.
  - For asymmetry, most of the systematic uncertainties cancelled, all data can be included to minimize the statistic error.
  - For cross section, the statistic uncertainty is less important, so only the data with small systematic uncertainty is selected.

\[
\Delta \sigma_\perp = A_\perp \times \sigma_0
\]

\[
A^{\text{phy}}_\perp = \frac{A^{\text{raw}}_\perp}{DP_b P_t} \quad A^{\text{raw}}_\perp = \frac{N^+}{Q^+} - \frac{N^-}{Q^-}
\]

\[
\sigma_0^{\text{phy}} = \sigma_0^{\text{raw}} \times D \quad \sigma_0^{\text{raw}} = \frac{N}{N_{\text{in}} \rho \epsilon_{\text{det}}} \times \frac{1}{A}
\]
Projections

\begin{align*}
\delta_{LT}(Q^2) &= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] \, dx \\
\int_0^1 g_2(x, Q^2) \, dx &= 0
\end{align*}

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Optics Goal

- The g2p experiment will measure the proton structure function $g_2$ in the low $Q^2$ region (0.02-0.2 GeV$^2$) for the first time
- Goal: 5% systematic uncertainty when measuring cross section
- Optics Goal:
  - <1.0% systematic uncertainty of scattering angle, which will contribute <4.0% to the uncertainty of cross section
  - $\sigma \sim 1/\sin^4(\theta/2)$
  - Momentum uncertainty is not as sensitive, but it is not hard to reach $10^{-4}$ level
Angle Calibration

- Determine the center scattering angle
  - Survey: ~1 mrad
  - Idea: Use elastic scattering on different target materials
    \[ \Delta E' = \frac{E}{1 + \frac{E}{M_1} (1 - \cos \theta)} - \frac{E}{1 + \frac{E}{M_2} (1 - \cos \theta)} \]
  - Data taking: Carbon foil in LHe, or CH\(_2\) foil
  - Two elastic peak took at the same time
  - The accuracy to determine this difference is <50 KeV -> <0.5 mrad
Matrix Calibration

- Calibrate the angle and momentum matrix elements:
  - Use carbon foil target and point beam
  - Use sieve slit to get the real scattering angle from geometry
  - Angle: Fit with data which we already know the real scattering angle
  - Momentum: Use the real scattering angle to calculate elastic scattering momentum of carbon target

Target
Sieve slit
Septa

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Matrix Calibration: Angle

LHRS Before Calibration

After Calibration

Resolution: 1.6 mrad (RMS)

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Matrix Calibration: Momentum

Before Calibration

After Calibration

LHRS

RMS: $1.5 \times 10^{-4}$
Matrix Calibration: Angle

Before Calibration

After Calibration

Resolution: 1.6mrad (RMS)
Matrix Calibration: Momentum

Before Calibration

Relative momentum

After Calibration

RMS: $1.7 \times 10^{-4}$
Optics Study with Target Field

- Recalibrate the angle matrix elements:
  - Start with the matrix without target field
  - To fit the matrix element, need to know the effective theta and phi angle
    - What we know is reaction point and the coordinate of the sieve hole
  - Trace the scattered electrons with different initial angles and select out the trajectory which goes through the sieve hole

\[ (x_{\text{ref}}, \theta_{\text{ref}}, y_{\text{ref}}, \phi_{\text{ref}}) \]
Optics Study with Target Field

- Reconstruct the scattering angle:
  - Use the HRS matrix to get the effective target variables
  - Project the effective target variables to sieve slit (red dot line)
  - Use the simulation package to calculate the trajectory of the scattered electron (red solid line), which will tell us the real scattering angle